

πNN coupling determined beyond the chiral limit

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Within the conventional QCD sum rules, we calculate the πNN coupling constant, $g_{\pi N}$, beyond the chiral limit using two-point correlation function with a pion. We consider the Dirac structure, $i\gamma_5$, at m_π^2 order, which has clear dependence on the PS and PV coupling schemes for the pion-nucleon interactions. For a consistent treatment of the sum rule, we include the linear terms in quark mass as they constitute the same chiral order as m_π^2 . Using the PS coupling scheme for the pion-nucleon interaction, we obtain $g_{\pi N} = 13.3 \pm 1.2$, which is very close to the empirical πNN coupling. This demonstrates that going beyond the chiral limit is crucial in determining the coupling and the pseudoscalar coupling scheme is preferable from the QCD point of view.

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QCD sum rule [1] is a framework which connects hadronic parameters with QCD parameters. In this framework, a correlation function is introduced in terms of interpolating fields constructed from quark and gluon fields. The interpolating field is constructed so that its coupling to the hadron of concern is expected to be strong while its couplings to other higher resonances are hoped to be small. Then the correlator is calculated by Wilson's operator product expansion (OPE) in the deep Euclidean region ($q^2 = -\infty$) and matched with the phenomenological "ansatz" to extract the hadron's information in terms of QCD parameters.

One interesting quantity to be determined is the pion-nucleon coupling constant, $g_{\pi N}$. As the coupling is empirically well-known, successful reproduction of this quantity may provide a solid framework to determine other meson-baryon couplings as well as a better understanding of nonperturbative nature of hadrons. For this direction, the two-point correlation function for the nucleon interpolating field J_N ,

$$\Pi(q, p) = i \int d^4x e^{iq \cdot x} \langle 0 | T [J_N(x) \bar{J}_N(0)] | \pi(p) \rangle, \quad (1)$$

may be useful and it is often used in calculating $g_{\pi N}$ [2–6]. Alternative approach is to consider the correlation function without pion but in an external axial field [7]. This provides the nucleon axial charge, g_A , which can be converted to $g_{\pi N}$ with the help of the Goldberger-Treiman relation. Our interest in this work is to provide a reasonable value of $g_{\pi N}$ using Eq. (1) because its extension to other meson-baryon couplings seems to be more straightforward. Further advantage in using Eq. (1) is to provide a criterion for the PS-PV coupling schemes for the pion-nucleon interaction as will be discussed below.

The correlation function, Eq. (1), contains various independent Dirac structures, each of which can be in principle used to calculate $g_{\pi N}$. For example, Ref. [4] uses the $\gamma_5 \not{p}$ structure while Ref. [3] uses the $i\gamma_5$ structure in the soft-pion limit. In the recent calculations [5,6], we proposed to use the $\gamma_5 \sigma_{\mu\nu}$ structure in studying $g_{\pi N}$ as this structure is independent of the pseudoscalar (PS) and pseudovector (PV) coupling schemes employed in the phenomenological side. This sum rule contains very small contribution from the transition $N \rightarrow N^*$, and the result is insensitive to the continuum threshold. Therefore, this structure provides a value of $g_{\pi N}$ independent of the coupling schemes. However, the result from this Dirac structure, $g_{\pi N} \sim 10$, is not quite satisfactory. Certainly a further improvement of this sum rule may be needed for the future extension to other SU(3) mesons.

Various improvements can be sought for. These may include a question related to the use of Ioffe's nucleon current for the correlator, higher order corrections in the OPE, or corrections associated with the chiral limit. The last possibility for the improvement is interesting because $g_{\pi N}$ from the $\gamma_5 \sigma_{\mu\nu}$ sum rule is rather close to the one in the chiral limit than its empirical value. In Ref. [5], the calculation is performed beyond the soft-pion limit by taking the leading order of the pion momentum p_μ , but for the rest of the correlator the chiral limit, $p^2 = m_\pi^2 = 0$, is taken. Thus, it is not clear whether the calculation is performed beyond the chiral limit and this may cause the discrepancy with the empirical $g_{\pi N}$.

In this paper, we pursue an improvement by presenting a QCD sum rule calculation beyond the chiral limit. Specifically, we consider the Dirac structure, $i\gamma_5$, at the order, $p^2 = m_\pi^2$. The sum rule for the structure, $i\gamma_5$, is, first of all, technically less involved when the calculation is done beyond the chiral limit. Secondly, even beyond the chiral limit, this structure is clearly PS-PV coupling-scheme dependent. Therefore, the successful

reproduction of the empirical value for $g_{\pi N}$ may provide an important QCD constraint for the pion-nucleon interaction type.

To see the coupling scheme dependence more clearly, we use the PS and PV Lagrangians

$$\mathcal{L}_{ps} = g_{\pi N} \bar{\psi} i \gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi} \psi ; \quad \mathcal{L}_{pv} = \frac{g_{\pi N}}{2m} \bar{\psi} \gamma_5 \gamma_\mu \boldsymbol{\tau} \cdot \partial^\mu \boldsymbol{\pi} \psi , \quad (2)$$

in constructing the phenomenological side of the correlator, Eq. (1). Then the correlator is expanded in terms of the pion momentum p_μ . Using the PS Lagrangian, we obtain for the $i\gamma_5$ structure [6],

$$g_{\pi N} \lambda^2 \left[-\frac{1}{q^2 - m^2} - \frac{p \cdot q}{(q^2 - m^2)^2} + \frac{p^2}{(q^2 - m^2)^2} \right] + \cdots . \quad (3)$$

Here λ is coupling of J_N to the physical nucleon, m is nucleon mass. Note that the first term is the phenomenological part of the sum rule in the soft-pion limit [3]. The second term containing $p \cdot q$ is not the same chiral order as m_π^2 . Thus at $p^2 = m_\pi^2$, the phenomenological correlator takes the form,

$$m_\pi^2 \frac{g_{\pi N} \lambda^2}{(q^2 - m^2)^2} + \cdots . \quad (4)$$

The ellipses indicate the contribution when J_N couples to higher resonances. This includes the continuum contribution whose spectral density is usually parameterized by a step function with a certain threshold S_π , and single pole terms associated with the transitions, $N \rightarrow N^*$ [8].

On the other hand, with the PV Lagrangian, the similar recipe yields the correlator at the order m_π^2 ,

$$\frac{m_\pi^2}{2} \frac{g_{\pi N} \lambda^2}{(q^2 - m^2)^2} + \cdots , \quad (5)$$

Note that in the PV case, there is no soft-pion limit as it should be. This PV correlator contains an additional residue of 1/2 compared to the PS correlator. Thus, $g_{\pi N}$ determined from the PV coupling scheme is twice of the one from the PS coupling scheme.

In the construction of this sum rule, the pion mass, m_π^2 , will be taken out as an overall factor. The rest correlator will be used to construct the sum rule. Then, a consistent treatment should be made also in the OPE side. Namely, from the Gell-Mann–Oakes–Renner relation,

$$-2m_q \langle \bar{q}q \rangle = m_\pi^2 f_\pi^2 , \quad (6)$$

the vanishing limit of the pion mass, $m_\pi^2 \rightarrow 0$, is consistent with the chiral limit, $m_q \rightarrow 0$. Therefore, for the sum rule with m_π^2 taken out as an overall factor, the quark-mass term should be kept in the OPE side. Clearly, this aspect has been overlooked in our previous calculations [6] and needs to be implemented.

To construct the OPE side, we consider the correlation function with a charged pion,

$$\Pi(q, p) = i \int d^4x e^{iq \cdot x} \langle 0 | T [J_p(x) \bar{J}_n(0)] | \pi^+(p) \rangle . \quad (7)$$

Here J_p is the proton interpolating field suggested by Ioffe [8],

$$J_p = \epsilon_{abc} [u_a^T C \gamma_\mu u_b] \gamma_5 \gamma^\mu d_c, \quad (8)$$

and the neutron interpolating field J_n is obtained by replacing $(u, d) \rightarrow (d, u)$. In the OPE, we keep the quark-antiquark component of the pion wave function and use the vacuum saturation hypothesis to factor out higher dimensional operators in terms of the pion wave function and the vacuum expectation value.

For the sum rule with the $i\gamma_5$ structure, we replace the quark-antiquark component of the pion wave function as follows,

$$\langle 0 | u_a^\alpha(x) \bar{d}_{a'}^\beta(0) | \pi^+(p) \rangle \rightarrow \frac{\delta_{aa'}}{12} (i\gamma_5)^{\alpha\beta} \langle 0 | \bar{d}(0) i\gamma_5 u(x) | \pi^+(p) \rangle. \quad (9)$$

At $p^2 = m_\pi^2$ order, the matrix element in the left-hand side is replaced as [9]

$$\langle 0 | \bar{d}(0) i\gamma_5 u(x) | \pi^+(p) \rangle \rightarrow -m_\pi^2 \frac{\sqrt{2} \langle \bar{q}q \rangle}{3f_\pi}, \quad (10)$$

where the overall normalization of the pion wave function at the second moment has been used. Another contribution at m_π^2 order is obtained by moving a gluon tensor from a quark propagator into the quark-antiquark component. This constitutes the three particle wave function whose overall normalization is relatively well-known. From Ref. [9],

$$\langle 0 | G_{\mu\nu}^A(0) u_a^\alpha(x) \bar{d}_b^\beta(0) | \pi^+(p) \rangle = -\frac{if_{3\pi}}{32} m_\pi^2 t_{ab}^A (\gamma_5 \sigma_{\mu\nu})^{\alpha\beta}, \quad (11)$$

where ¹ $f_{3\pi} = 0.003 \text{ GeV}^2$ and the color matrices t^A are related to the Gell-Mann matrices via $t^A = \lambda^A/2$.

As we have discussed, the linear terms in quark mass should be kept in the OPE for the sum rule at m_π^2 order. The quark-mass dependent terms can be obtained by first taking the limit, $p_\mu \rightarrow 0$, in the quark-antiquark component, Eq. (9), while picking up linear terms in quark-mass from the other part of the correlator ². It turns out that the condensates, $m_q \langle \bar{q}q \rangle$ and $m_q \langle \bar{q}g_s \sigma \cdot \mathcal{G}q \rangle \equiv m_q m_0^2 \langle \bar{q}q \rangle$, contribute to the OPE of the $i\gamma_5$ structure. The Gell-Mann–Oakes–Renner relation is used to convert $m_q \langle \bar{q}q \rangle$ to $-m_\pi^2 f_\pi^2/2$. Therefore, the quark-mass terms give additional contributions to the sum rule at m_π^2 order.

Collecting all the OPE terms contributing to the $i\gamma_5$ structure at m_π^2 order, the OPE side (after taking out the isospin factor $\sqrt{2}$ as well as m_π^2 as overall factors) takes the form

¹Its value is uncertain by an error $\pm 0.0005 \text{ GeV}^2$ depending on the renormalization scale [7]. However, the contribution from Eq. (11) is small in our sum rule as we will discuss below. Thus, this error in $f_{3\pi}$ is negligible in our sum rule.

²For complete quark propagator including the linear order in quark mass, see Ref. [10]. Note that gluonic tensor used there has opposite sign of that in Ref. [2]. This is just a matter of how one defines the covariant derivative but, in practice, this sign difference should be carefully noted.

$$\ln(-q^2) \left[\frac{\langle \bar{q}q \rangle}{12\pi^2 f_\pi} + \frac{3f_{3\pi}}{4\sqrt{2}\pi^2} \right] + f_\pi \langle \bar{q}q \rangle \frac{1}{q^2} + \frac{1}{72f_\pi} \langle \bar{q}q \rangle \left\langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \right\rangle \frac{1}{q^4} - \frac{1}{3} m_0^2 f_\pi \langle \bar{q}q \rangle \frac{1}{q^4} \quad (12)$$

Note, we use the pion decay constant $f_\pi = 0.093$ GeV here. The second and fourth terms come from the quark-mass dependent terms. It turns out that these are important in stabilizing the sum rule, justifying the inclusion of quark-mass terms in the OPE. The second term in the bracket comes from gluonic contribution combined with the quark-antiquark component, Eq. (11). Its contribution is about 4 times smaller than the first term in the bracket. Except for this term, all others contain the quark condensate. This feature provides very stable results when this sum rule combined with the nucleon chiral-odd sum rule.

We now match the OPE with its pseudoscalar phenomenological part, Eq.(4). To saturate the correlator with the low-lying resonance, we perform Borel transformation and obtain,

$$g_{\pi N} \lambda^2 e^{-m^2/M^2} [1 + AM^2] = -M^4 E_0(x_\pi) \left[\frac{\langle \bar{q}q \rangle}{12\pi^2 f_\pi} + \frac{3f_{3\pi}}{4\sqrt{2}\pi^2} \right] - f_\pi \langle \bar{q}q \rangle M^2 + \frac{1}{72f_\pi} \langle \bar{q}q \rangle \left\langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \right\rangle - \frac{1}{3} m_0^2 f_\pi \langle \bar{q}q \rangle. \quad (13)$$

The contribution from $N \rightarrow N^*$ [8] is denoted by the unknown constant, A . The continuum contribution is included in the factor, $E_n(x_\pi \equiv S_\pi/M^2) = 1 - (1 + x_\pi + \dots + x_\pi^n/n!)e^{-x_\pi}$ where S_π is the continuum threshold, which we take 2.07 GeV^2 corresponding to the Roper resonance. In our analysis, we take standard values for the QCD parameters,

$$\langle \bar{q}q \rangle = -(0.23 \text{ GeV})^3; \quad \left\langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \right\rangle = (0.33 \text{ GeV})^4; \quad m_0^2 = 0.8 \text{ GeV}^2. \quad (14)$$

In figure 1, we plot $g_{\pi N} \lambda^2 [1 + AM^2]$ as a function of the Borel mass M^2 . To see the sensitivity to the continuum threshold, we also plot the curve with $S_\pi = 2.57 \text{ GeV}^2$, which is very close to the one with $S_\pi = 2.07 \text{ GeV}^2$. The two curves differs only by 2% at $M^2 = 1 \text{ GeV}^2$, indicating that our sum rule is insensitive to the continuum threshold. The highest dimensional term in the OPE contributes appreciably for $M^2 \leq 0.6 \text{ GeV}^2$, more than 20 % of the total OPE. Thus, the truncated OPE therefore will be reliable for $M^2 \geq 0.6 \text{ GeV}^2$. The slope of the curve for $M^2 \geq 0.6 \text{ GeV}^2$ is small, indicating that the unknown single pole term denoted by A is small.

To eliminate the dependence on the unknown strength λ in our sum rule, we divide Eq. (13) by the nucleon chiral-odd sum rule and obtain,

$$\begin{aligned} \frac{g_{\pi N}}{m} [1 + AM^2] = & \left\{ M^4 E_0(x_\pi) \left[\frac{1}{3f_\pi} + \frac{3f_{3\pi}}{\langle \bar{q}q \rangle \sqrt{2}} \right] + \frac{8\pi^2 f_\pi}{3} M^2 - \frac{\pi^2}{18f_\pi} \left\langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \right\rangle + \frac{4\pi^2}{3} m_0^2 f_\pi \right\} \\ & \times \left\{ M^4 E_1(x_N) - \frac{\pi^2}{6} \left\langle \frac{\alpha_s}{\pi} \mathcal{G}^2 \right\rangle \right\}^{-1}, \end{aligned} \quad (15)$$

where $x_N = S_N/M^2$ with S_N being the continuum threshold for the nucleon sum rule. Note that the dependence on the quark condensate has been mostly canceled in the ratio, leaving a slight dependence in the term $f_{3\pi}$. Additional source of the uncertainty associated with the gluon condensate is also very small as it is canceled in the ratio. One important uncertainty

comes from the parameter m_0^2 , which however appears only in the highest dimensional OPE. Therefore, its contribution will be suppressed in the Borel window chosen. The error from QCD parameters is estimated numerically and it is about ± 1.2 in determining $g_{\pi N}$. For the continuum threshold in the nucleon sum rule, we take $S_N = S_\pi$. This choice is made because at the chiral limit the $i\gamma_5$ sum rule is equivalent to the nucleon chiral-odd sum rule; these two are related by chiral rotation [4]. This equivalence provides the Goldberger-Treiman relation with $g_A = 1$ [3]. This choice for the continuum is also supported from modeling higher resonance contributions to the correlator based on effective models [6]. We determine $g_{\pi N}$ and A by fitting the RHS with a straight line within the appropriately chosen Borel window. The dependence on the Borel mass is mainly driven by the nucleon sum rule. The maximum Borel mass is determined by restricting the the continuum contribution from the nucleon sum rule while the minimum Borel mass is obtained by restricting the highest OPE term from the πNN sum rule. These gives the common window of the two sum rules, $0.65 \leq M^2 \leq 1.24$. By fitting the RHS with a straight line within this window, we obtain $g_{\pi N} = 13.3 \pm 1.2$, where the quoted error comes from the QCD parameters. This is remarkably close to its empirical value of 13.4.

In getting this result, it is essential to go beyond the chiral limit. Since the empirical $g_{\pi N}$ should include the chiral corrections, it is indeed natural to go beyond the chiral limit in the determination of $g_{\pi N}$. One important observation made in this work is that the quark-mass dependent terms shouldn't be treated separately from the sum rule proportional to m_π^2 as they constitute the same chiral order via the Gell-Mann–Oakes–Renner relation. The quark-mass terms are found to be very important in stabilizing the sum rule. Our findings, remarkable agreement with the empirical value and the insensitiveness on the QCD parameters, may provide a solid ground in constructing sum rules for other meson-baryon couplings. The predictive power of QCD sum rules can be substantially enhanced. One application of our sum rule to the ηNN coupling is in progress [11]. Furthermore, our result provides a QCD constraint for the type of the pion-nucleon coupling. The value of $g_{\pi N}$ quoted above is based on the PS coupling scheme. With the PV coupling scheme, we would have obtained the value twice of the quoted above. [See Eq.(5).] Any error in our approach can not produce the value of $g_{\pi N}$ consistent with the PV coupling scheme. Therefore our work suggests that the PS scheme is preferable for the pion-nucleon coupling from the QCD point of view.

In summary, we have developed a QCD sum rule for πNN coupling beyond the chiral limit for the first time. The quark-mass dependent terms are combined to the sum rule proportional to m_π^2 and they are very important in this sum rule. A remarkable agreement with the empirical value of $g_{\pi N}$ was obtained with very small errors. This sum rule provides the first QCD constraint for the type of the pion-nucleon coupling, in favor of the pseudoscalar coupling.

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FIGURES

FIG. 1. The Borel mass dependence of $g_{\pi N} \lambda^2 [1 + AM^2]$. The solid line is for $S_\pi = 2.07 \text{ GeV}^2$ and the dashed line is for $S_\pi = 2.57 \text{ GeV}^2$. The two curves are differed only by 2% at $M^2 = 1 \text{ GeV}^2$.

